Main results:

The section number is correspondent to the section number in the report.

# 1 Introduction

## 2 Localization

In this chapter we thoroughly studied the Cramer-Rao bound on the localization problem in both anchored (3 or more nodes with known positions) case and anchor-free localization case. Then we proposed a 2-step anchored localization scheme.

## 2.1 Problem Formulation

## 2.2 CR bounds

## 2.2.1 Anchored localization:

1) We gave an explicit expression of the Fisher Information Matrix solely based on the geometry of the sensor network.

2) We showed that the Cramer Rao bound is invariant under zooming and translation. Rotation does not change the lower bound on  $E((x - \hat{x})^2) + (y - \hat{y})^2)$ .

3) By matrix theory, we gave a lower bound on the Cramer-Rao bound. This lower bound can be computed using only local geometry. And it converges to CR bound if the local area is expanded.

#### 2.2.2 Anchor-free localization:

1)We proposed the equivalence class estimation concept due to the singularity of the Fisher Information Matrix for anchor-free localization.

2)A Cramer-Rao-like bound is presented and it is invariant under zooming, translation and rotation.

3) We observe that per node bound is only dependent on the average number of neighbor nodes. We conjecture that the average estimation variance mostly depends on received signal power per node.

#### 2.3 2-step localization scheme

We proposed a 2 step anchored localization scheme. In the first step we estimate the anchor-free coordinate systems around individual nodes. In the second step we combine all the anchor-free coordinate systems together. And with the anchor information, we can assign anchored coordinate

to each node. The novelty of this algorithm is that we combine 2 anchor-free coordinate systems through the *optimal* (MSE sense) rigid translation.

### 2.4 synchronization

We proposed a centralized linear synchronization scheme which achieves the lower bound with the assumption that the noises are iid Gaussian.

# 3 Tracking Objects by a Sensor Network

In this chapter, we studied the Cramer-Rao bound on object(s) tracking by a sensor network and proposed several tracking schemes.

## 3.1 Single Tx, Single Rx

1) Without motion model, using only multipath distance measures cannot track the object because two different motions could yield same multipath distances measures all the time..

2) Theoretically, if an object moves with a strict constant velocity motion, we can estimate the motion of it in a single Tx, single Rx network.

3) We calculate the Cramer-Rao bound and it is too large to be good.

4) We proposed a consistent estimation scheme that the estimation converges to the true value when the number of measurements goes to infinity.

## 3.2 Multiple Tx, Multiple/Single Rx, Single Object

## 3.2.1 CR bound for multiple Tx, multiple Rx

We computed the asymptotic CR bound as the number of Tx's, N, and the number of Rx's, M, increase to infinity. The CR bound is  $\sim \frac{1}{NM}$  if the object is inside the sensor field, with the uniform distribution assumption on the positions of the sensors.

#### 3.2.2 A position estimation scheme

We proposed a semi-linear position estimation scheme, which is order optimal comparing with the Cramer-Rao bound.

#### 3.2.3 CR bound for multiple Tx, single Rx

With similar assumptions for multiple Rx case, we computed the asymptotic CR bound as the number of Tx's, N, increase to infinity. The CR bound is  $\sim \frac{1}{N}$ .

#### 3.2.4 CR bound for a faraway object

We studied the asymptotic property of the Cramer-Rao bound if the object is far away from the sensor field and N, M are big. In the Euclidean coordinate the Cramer-Rao bound  $\sim \frac{(L/r)^2}{NM}$ , where L is the distance between the object and the sensor field meanwhile r is the radius of the sensor field. In the contrary, in the polar coordinate,  $E((\hat{\rho} - \rho)^2) \sim 1$  and  $E((\hat{\theta} - \theta)^2) \sim 1$ . This suggests that we can still accurately estimate the communication channel.

#### 3.2.5 Position estimation in a small sensor network

We proposed a simple semi-linear position estimation scheme which works well if the SNR is reasonably high. We studied the 2Tx, 2 Rx sensor network thoroughly and found that the sensor placement problem is likely to be very interesting for a small sensor network by showing the huge performance difference between two different sensor displacements.

## 3.3 Multiple Tx, Multiple Rx, Multiple Object

The difficulty for multiple objects tracking is that it is hard to associate multipath distance measures with the objects.

1) We first showed that exhausting all the possible associations needs exponential computation loads which makes the exhaustive scheme impractical.

2) We proposed a 2-step algorithm. First using a hough-transform-like algorithm to associate multipath-distance-measures with the objects. Then we use the semi-linear single object tracking algorithm to give a final position estimation. The computational load is polynomial. We show that the problem is difficult and present a practically working algorithm.